Tutorial 4 for MATH 2020A (2024 Fall)

1. Let $\Omega \subset \mathbb{R}^3$ be the region common to the interiors of the cylinders $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$.

   (a) Sketch the region Ω in the first octant.

   (b) Find its volume.

   Solution: (b)$\frac{16}{3}$

2. Let $R \subset \mathbb{R}^3$ be the cube bounded by the coordinate planes and the planes $x = 2$, $y = 2$, and $z = 2$. Find the average value of $F(x, y, z) = x^2 + 9$ over R.

   Solution: $\frac{31}{3}$

3. Let $R \subset \mathbb{R}^2$ be the thin plate bounded by the parabola $x = y - y^2$ and the line $x + y = 0$.

   (a) Draw the thin plate R in $xy-$plane and label all the intersects.

   (b) If the density of this plate is given by $\delta(x, y) = x + y$, find the moment of inertia about the $x-$axis of R.

   Solution: I apologize for a mistake on Monday's tutorial, the formula for moment of inertia of a $\mathbb{R}^2-$region R about the $x-$axis should be

   $$I_x = \iint_R y^2 \delta(x, y) \, \mathrm{d}A,$$

   and the correct answer for (b) should be $\frac{64}{105}$. Thanks a lot for the correction!

4. Let $\Omega \subset \mathbb{R}^3$ be the solid between the sphere $S_1 : \rho = \cos\phi$ and the hemisphere $S_2 : \rho = 2, 0 \leq \phi \leq \frac{\pi}{2}$(all in spherical coordinate).

   (a) Sketch S_1 and S_2.

   (b) Represent the solid Ω in spherical coordinate and find its volume.

   Solution: (b) $\frac{31\pi}{6}$

5. Let $\Omega \subset \mathbb{R}^3$ be the smaller cap cut from a solid ball of radius 2 by a plane with distance 1 from the center of the sphere. Express the volume of Ω as an iterated triple integral in

   (a) Cartesian coordinate;

   (b) Cylindrical coordinate;

   (c) Spherical coordinate.

   Then find the volume of Ω.

   Solution: $V(\Omega) = \frac{5\pi}{3}$